Exercises (week 6)

Question 1 Let

\[ A = \begin{pmatrix}
1 & 2 & 1 & -1 \\
1 & 5 & 0 & 2 \\
0 & 3 & -1 & 3 \\
1 & -4 & 3 & -7
\end{pmatrix} \]

1. Compute the row-reduced echelon form (RREF) of \( A \) using Gauss-Jordan elimination.
2. Find a basis for \( \text{col}(A) \) and \( \text{row}(A) \).
3. What is the rank of \( A \)? Deduce a basis for \( \text{null}(A) \).

Question 2 Let \( A \in \mathbb{R}^{n \times n} \) be an invertible matrix. Determine the row and column spaces of \( A \).

Question 3 True/False: The rank of \( A \) is equal to the number of zero rows in the RREF of \( A \)

1. for \( A \in \mathbb{R}^{n \times n} \)
2. for \( A \in \mathbb{R}^{m \times n} \) with \( m < n \)
3. for \( A \in \mathbb{R}^{m \times n} \) with \( m > n \)

Question 4 (problem 3 from the Textbook, chapter VII) True/False: For any \( A, B \in \mathbb{R}^{n \times n} \), \( \text{rank}(A + B) > \text{rank}(A) + \text{rank}(B) \).

Question 5 (problem 5 from the Textbook, chapter VII) True/False: If the RREF of \( A \in \mathbb{R}^{m \times n} \) has at least one zero row, then \( \text{dim}(\text{null}(A)) > 0 \). Hint: consider three cases, that is, \( m = n \), \( m > n \) and \( m < n \).