Exercises (week 10)

Question 1 Consider $A = \begin{bmatrix} r_1 \\ \vdots \\ r_{10} \end{bmatrix} \in 10 \times 10$ and define $B \in 10 \times 10$ from $A$ as $B = \begin{bmatrix} r_1 \\ \vdots \\ r_6 \\ r_7 + 6r_5 \\ r_8 \\ \vdots \\ r_{10} \end{bmatrix}$. What can be said about $\det_{10}(B)$?

Question 2 True/False:

1. For any $A, B \in n \times n$, $\det_n(A + B) = \det_n(A) + \det_n(B)$.
2. For any $A \in n \times n$ and $\lambda \in \mathbb{R}$, $\det_n(\lambda A) = \lambda \det_n(A)$.
3. For any $x, y \in n \times n$, $\det(xy^T) = 0$.

Question 3 Compute the determinant of the $n \times n$ matrix

$$
\begin{pmatrix}
1 & 1 & \ldots & 1 \\
1 & 1 & \ldots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
1 & 1 & \ldots & 1
\end{pmatrix}
$$

Question 4 Let $A \in n \times n$ be a matrix of integers such that any entry of the $i$-th row is divisible by $i$ (for $i = 1, 2, \ldots, n$). Prove that $\det_n(A)$ is divisible by $n$!

Question 5 Consider 4 points $(x_i, y_i, z_i) \in \mathbb{R}^3$, $i = 1, \ldots, 4$. Prove that these points belong to the same plane if and only if

$$
\det_4 \begin{pmatrix}
x_1 & y_1 & z_1 & 1 \\
x_2 & y_2 & z_2 & 1 \\
x_3 & y_3 & z_3 & 1 \\
x_4 & y_4 & z_4 & 1
\end{pmatrix} = 0.
$$